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Sector-focused stability methods for robust source localization in matched-field processing

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Capon's maximum likelihood method (MLM) is being used increasingly for improved localization and high resolution of sources in matched-field processing (MFP) in a waveguide environment. When the noise component is dominated by modal noise (that is, noise due to excitation of the modal structure of the waveguide by distant sources of acoustic energy, e.g., wave action, distant shipping), the MLM can become unstable; when the number of modes (M) supported by the waveguide is considerably less than the number of sensors (N), a "reduced" MLM is available to stabilize the processing [Byrne *et al.*, J. Acoust. Soc. Am. **87**, 2493–2502 (1990)]. In this paper the sector-focused stability (SFS) methods of Byrne and Steele (Proc. IEEE., ICASSP 1987) are employed to treat the case of M nearly equal to N. Simulations, using a shallow-water range-independent environment, are presented to illustrate the increase in stability of SFS over MLM in the presence of phase errors on the sensors or more general mismatch.

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INTRODUCTION

Matched-field processing (MFP) is a type of signal processing procedure that involves the comparison of signal replica vectors obtained from models of the acoustic propagation, such as normal mode expansions for range-independent waveguides, with received signal data. In cases where a plane-wave propagation model is generally inadequate, MFP has been shown to be a useful method for detection and localization of sources. The need to go beyond the plane-wave assumption was discussed more than a decade ago by Hinich[1] and Bucker,[2] and later by Mermoz.[3] Tindle *et al.*[4] introduced what has come to be called "mode space processing." Hinich[5] considered modal noise as a superposition of sources in the waveguide and showed that the statistical maximum likelihood estimation of unknown parameters could be achieved with the number of sensors (N) equal to the number of modes (M) supported by the waveguide. Heitmeyer *et al.*[6] considered MFP for the Pekeris model.

To achieve high resolution, while controlling computational load, several authors have considered nonlinear methods. The most widely studied is Capon's estimator,[7] called the "maximum likelihood method" (MLM) by Lacoss,[8] as well as the "minimum variance distortionless look" by others.[9] For a detailed discussion of MLM applied to MFP see Baggeroer *et al.*[10] Klemm[11,12] considered a maximum entropy-type estimator and studied the effects of mismatch due to incomplete or inaccurate knowledge of environmental parameters, such as those describing the bottom. Other studies of sensitivity of MFP to mismatch have considered: perturbations in water depth and geoacoustic parameters;[13] the use of waterborne modes only, as well as sound-speed profile and sediment mismatch;[14] and reduction of array aperture and coverage of the water column.[15] Several authors have investigated the estimation procedures themselves and have suggested new estimators to overcome problems with mismatch. Shang[16–18] has studied the mode space processing mentioned in Tindle *et al.*,[4] and Yang[19] has suggested projection onto certain eigenvector subspaces to increase the number of resolvable modes. Shang[20] has also discussed the use of MFP with known sources to determine waveguide parameters.

It is well known that Capon's MLM estimator is sensitive to mismatch between the assumed model and the actual data. For the plane-wave, case Byrne and Steele[21] showed that certain eigenvectors of the cross spectral matrix (CSM) R could become unstable when a correlated noise component is present, which leads to deterioration of the MLM in the presence of phase errors or other mismatch. A "sector-focused stability" (SFS) method has been developed[22] to overcome this instability. The situation in the normal mode, rather than the plane-wave, case is closely analogous to that studied by Byrne and Steele.[21,22] The presence of modal noise (distant noise sources exciting the modal structure) creates similar eigenvector instability. Byrne *et al.*[23] derived mode space MLM, or "reduced ML" (RML), as a special case of SFS and showed that it could improve stability when

the number of modes is significantly less than the number of phones. In this paper we develop sector focusing further, to achieve increased stability and reduced computational load in the normal mode case. This enhanced sector to agree with focusing can improve RML when the number of modes is about equal to the number of phones.

In Sec. I we present some background on the issue of sensitivity to mismatch of ML and on the recently popular technique of dimensionality reduction, of which SFS is a special case. In Sec. II we present the SFS method in detail, discuss the normal mode model, and consider the role of normalization in matched-field processing. In Sec. III we discuss the simulations performed and describe the results. Finally, we have a brief statement of the conclusions we draw from this work.

I. BACKGROUND

Shortly after the appearance of Capon's paper,[7] Seleg-son[24] showed that the resolving properties of the MLM estimator could deteriorate if there are phase or amplitude errors in the data. He also noted (in a private communication recalled by McDonough[25]) that the wider the spread between the largest and smallest eigenvalues of the cross spectral matrix R, the more severe the deterioration. McDonough[25] obtained quantitative measures of the sensitivity of MLM to mismatch. Cox[26,27] viewed sensitivity to mismatch as related to ability to resolve sources or to reject interference, and suggested several methods to improve stability, including multiple main lobe constraints. Vural[28] pointed out that mismatch is most serious at high signal-to-noise ratio (SNR) and considered the use of multiple main lobe constraints and derivative constraints to combat the problem. The use of multiple linear constraints was also treated by Steele.[29] Gray[30] noted that stability can be obtained by processing the output of several presteered beams (beam space processing), provided the beams are sufficiently orthogonal. More recently, Cox *et al.*[31] have achieved robust beamforming by imposing a constraint on white noise response.

Byrne and Steele[21] related the sensitivity of the MLM estimator to that of particular eigenvectors of the CSM and showed that, by modifying MLM to reduce the influence of the unstable eigenvectors, stable estimation was possible. Their analysis showed that instability increases with the presence of noise components that appeared signal-like to the array (such as spherical isotropic noise to an oversampled line array). Of course, such components lead to the eigenvalue spread that concerned Seligson and McDonough. To combat sensitivity, Byrne and Steele[22] suggested the SFS procedure, which also has the advantage of dimensionality reduction, reducing the size of the matrix to be inverted for estimation by using an N by K ($K<N$) projector matrix V, with $V^\dagger V=I$ (the $\dagger$ denotes conjugate transpose), to transform R to the K by K matrix $V^\dagger R V$. Dimensionality reduction has become popular recently and has been used for a variety of purposes, including simple reduction of computational load.

Several authors have considered dimensionality reduction for the plane-wave case. Bienvenu and Kopp[32] suggested beam space processing to avoid the effects of nonwhite noise on eigenvector-based methods. Lee and Wengrovitz[33] showed that the SFS procedure can be effective in overcoming mismatch due to finite averaging in estimating the CSM. Van Veen and Williams[34] considered general dimensionality reduction methods for reducing computation, and Xu and Buckley[35] analyzed the statistical improvement due to dimensionality reduction. Cox *et al.*[36] have studied conventional beamforming on selected subarrays for MFP with long arrays, to reduce computational load.

In matched-field processing one is usually dealing with a waveguide that imposes structure on the propagating field. Distant sources of acoustic energy (shipping, surface waves) can excite the modal structure of the waveguide and present to the array a noise vector whose structure reflects that waveguide. We refer to this as modal noise. When the waveguide supports fewer modes than phones ($M<N$), such modal noise presents to the array the same sort of structured, low-dimensional vectors presented by signal sources. The problem is closely analogous to the spherical isotropic noise and oversampled array case discussed above.[22]

Dimensionality reduction can be implemented in MFP through so-called mode space processing (MSP). In a related paper, Byrne *et al.*[23] used the SFS approach to derive mode space MLM processing and to examine the improvement in stability this procedure affords. When the number of modes is not significantly less than the number of sensors the advantage of MSP over MLM is not dramatic, and further reduction of dimension is warranted. We consider a method for achieving further dimensional reduction in this paper: using regions of range-depth space to define the sectors; and using the eigenstructure of related positive definite matrices to determine the essential dimensions of the sectors, and to provide the transformations required.

The basic idea of dimensionality reduction is the use of a nonsquare N by K matrix V ($K<N$) with orthonormal columns ($V^\dagger V=I$) to project the data vectors onto a subspace of dimension K. The resulting CSM of the projected data is $T=V^\dagger R V$, which is K by K. All subsequent processing involves only T. If the columns of V are the steering vectors associated with pre-selected beams, then T is the CSM used in beam space processing.[30] If the beams are not orthogonal initially, an orthogonalization procedure (using, say, "QR factorization" or Gram–Schmidt[37]) is needed to produce V. If the columns of V are selected columns of the N by N identity matrix, then T is the CSM obtained by deleting certain sensors of the array (subarray processing). One objective of dimensionality reduction is to reduce the size of the matrix to be inverted without destroying much signal information. If the signal vector of interest lies in the span of the columns of V, then there is no loss of signal energy. Another objective is to prewhiten the noise-only CSM. If $R=S+Q$, with S and Q the signals-only and noise-only CSM, respectively, then to whiten Q with a square matrix (no dimensionality reduction) requires exact knowledge of Q. However, $V^\dagger QV$ can be made close to a K by K identity matrix, when $K \ll N$ and the columns of V are obtained from nearby beam-steering vectors, that is, we can achieve a local whitening of the noise without knowing Q.[33]

When the objective is to stabilize MLM estimation, the projection matrix V is selected to eliminate or reduce the contribution coming from unstable eigenvectors of R. For the case of normal mode propagation considered in this paper, the columns of V can be chosen to be the M mode-sampling vectors (leading to the RML estimator[23]) or can correspond to a subsector of range and depth space.

II. THE NORMAL MODE MODEL, SECTOR-FOCUSED STABILITY AND NORMALIZATION

A. The normal mode model

Using the notation of Shang[16] we write the field excited by a point source in the far field of a waveguide in a normal mode expansion as

$$p(r,z,z_0) = \pi i \sum_{m=1}^{M} U_m(z) s_m(r,z_0), \tag{1}$$

where z_0 is the source depth, z is the receiver depth, and r is the source range from an arbitrary origin (usually where the vertical array is located). The $U_m(z)$ are the eigenfunctions of the depth-only Sturm–Liouville boundary value problem: $U_m(z)$ is called the mth mode. The coefficients $s_m(r,z_0)$ represent the excitation of the mth mode by a source at range r and depth z_0:

$$s_m(r,z_0) = \exp(3\pi i/4)\exp(-\beta_m r)\exp(ik_m r) \times U_m(z_0)\sqrt{2\pi/k_m r}, \tag{2}$$

where k_m and β_m are horizontal and vertical wave numbers, respectively. Higher modes and continuous modes are considered as noise, since we are interested in far-field sources. Sampling the pressure field using sensors at depths z_n $(n = 1,...,N)$ we obtain the (narrow-band) data vector

$$\mathbf{p}(r,z_0) = [p(r,z_1,z_0),...,p(r,z_N,z_0)]^T. \tag{3}$$

Letting U be the N by M matrix whose mth column is $[(U_m(z_1),...,U_m(z_n)]^T$, and $\mathbf{s}(r,z_0)$ be the M by 1 vector whose mth entry is $s_m(r,z_0)$, we can then write

$$\mathbf{p}(r,z_0) = U\mathbf{s}(r,z_0). \tag{4}$$

We assume that the single snapshot data vector $\mathbf{x}$ has the form

$$\mathbf{x} = \sum_{j=1}^{J} A_j \mathbf{p}(r_j,z_0 j) + \text{noise} = \sum_{j=1}^{J} A_j U\mathbf{s}_j + U\gamma + \eta, \tag{5}$$

where r_j and z_{0_j} are the range and depth, respectively, of the jth source; $\mathbf{s}_j = \mathbf{s}(r_j,z_{0_j})$; the entries of γ represent the excitation of the various modal amplitudes by aggregate noise sources; η is random white noise; and the A_j are independent random variables representing time-varying channel fluctuations (a Rayleigh fading channel, for example). To lessen the effects of noise it is common to take $R = \text{average}(\mathbf{xx}^\dagger)$, where the averaging is performed over the available snapshots. Then our model for R becomes

$$R = USAS^\dagger U^\dagger + UGU^\dagger + H, \tag{6}$$

where the jth column of S is $\mathbf{s}(r_j,z_{0_j})$; A is the variance–covariance matrix of the A_j; $UGU^\dagger$ is the CSM of the modal noise (so $G = \text{average}(\gamma\gamma^\dagger)$ and has the same form as $SAS^\dagger$, but for a large J, representing the contributions of many distant noise sources); and $H = \text{average}(\eta\eta^\dagger)$ is the CSM of nonmodal noise.

When the number of modes M is less than the number of sensors N, both the signal vectors $U\mathbf{s}_j$ and the modal noise vectors $U\gamma$ represent energy projected onto a smaller dimensional subspace, the span of the columns of U, so that signals and modal noise are not easily separated. When the modal noise component $UGU^\dagger$ dominates R, the lowest eigenvectors of R tend to be nearly orthogonal to all the columns of U and hence to every $\mathbf{p}(r,z_0) = U\mathbf{s}(r,z_0)$. This is closely analogous to the plane-wave situation, with spherical isotropic noise and an oversampled linear array, and will cause MLM to be sensitive to phase errors.

If M is much less than N, stability can be achieved by letting the V in the SFS method be the (orthogonalized) columns of U.[21] If M is not much less than N, greater stability can be obtained by choosing as our sector a subset Σ of $\Omega = (r,z)$ space whose essential dimension is $K < M$, and then proceeding with SFS on the reduced K by K matrix $V^\dagger RV$, as described below.

In Sec. III we discuss simulated cases of normal mode propagation and the use of SFS to stabilize the MLM estimator.

B. Sector-focused stability

We assume, initially, an arbitrarily configured array of N sensors. The narrow-band single-snapshot data vector is the N by 1 vector $\mathbf{x}$ and the (sampled) cross spectral matrix is $R = \text{average}(\mathbf{xx}^\dagger)$, where the average is taken over the available number of snapshots. We assume that R has (approximately) the form

$$R = \sum_{j=1}^{J} \alpha_j^2 \mathbf{p}(\theta_j)\mathbf{p}(\theta_j)^\dagger + Q, \tag{7}$$

where the $\mathbf{p}(\theta_j)$ are the signal vectors assumed to lie in a parametrized set $\{\mathbf{p}(\theta), \theta \text{ in } \Omega\}$ and Q is the noise-only CSM. For now the θ have no further significance and may be scalar or vector quantities. The objective is to determine J, as well as the θ_j and (perhaps) the α_j^2, usually without knowledge of Q. Capon's MLM estimator is a popular choice because it does not require Q and, for sufficiently high signal-to-noise ratio (SNR), achieves better resolution (determination of J) than conventional methods (e.g., Bartlett).[7,8] Other procedures, such as "optimal array processing" or eigenvector-based methods, require a prewhitening of R that necessitates knowledge of Q.

Capon's MLM proceeds as follows: for each fixed value $\theta = \omega$ we find the linear filter $\mathbf{f} = \mathbf{f}(\omega)$ that minimizes average output power $\mathbf{f}^\dagger R\,\mathbf{f}$, subject only to the constraint of no distortion of $\mathbf{p}(\omega)$: $f^\dagger\mathbf{p}(\omega) = 1$. The well-known solution is

$$\mathbf{f}(\omega) = R^{-1}\mathbf{p}(\omega)/[\mathbf{p}(\omega)^\dagger R^{-1}\mathbf{p}(\omega)]. \tag{8}$$

Applying this optimal filter to the actual data gives the average output power $\mathbf{f}^\dagger(\omega) R\,\mathbf{f}(\omega)$ equal to

$$1/[\mathbf{p}(\omega)^\dagger R^{-1}\mathbf{p}(\omega)] = \text{MLM}(\omega), \tag{9}$$

which is Capon's estimate of the power associated with $\theta = \omega$. Because of the appearance of R^{-1} in (9), those ei-

genvectors of R corresponding to the lowest eigenvalues (let us call them the "lowest" eigenvectors) have the greatest influence on MLM(ω). As discussed by Byrne and Steele,[21] the presence of a correlated noise component Q, corresponding to the superposition of many signal-like noise sources, can cause these lowest eigenvectors to become unstable in the presence of mismatch (such as phase errors). To stabilize the estimation of the θ_j one must reduce or eliminate the contribution from these unstable eigenvectors. The SFS method is designed to achieve this stability.

The MLM estimator is the averaged output of the linear filter in (8), which is dominated by the lowest eigenvectors of R. When the modal noise ($UGU^\dagger$) component of Q is significant, the lowest eigenvectors of R tend to be nearly orthogonal to the columns of U, hence to all $p(\theta)$, and MLM becomes unstable. The sector-focused stability (SFS) method employs a modification of (8) to eliminate the instability, at least for estimation within a pre-selected subset Σ of Ω.

For any N by K matrix V, every linear filter $\mathbf{g} = \mathbf{g}(\omega)$ can be written uniquely as $\mathbf{g} = V\mathbf{a} + \mathbf{w}$, where $V^\dagger \mathbf{w} = 0$ and $\mathbf{a}$ is some K by 1 vector. The more $\mathbf{w}$ dominates $\mathbf{g}$ the more $\mathbf{g}$ will be orthogonal to the columns of V. Those $\mathbf{g}$ whose $\mathbf{w}$ component is missing are those least orthogonal to the columns of V; thus the additional constraint that $\mathbf{g}(\omega) = V\mathbf{a}$ for some $\mathbf{a}$ simply makes it harder for $\mathbf{g}(\omega)$ to be orthogonal to the columns of V. The answer is to build V from vectors $\mathbf{p}(\theta)$, θ in Σ, thus making $g(\omega)$ more stable for estimation within Σ.

To obtain the projection matrix V we begin by simulating a noise component equal to the superposition of many independent sources within Σ; that is, we form (an approximation of) $Q(\Sigma) = \text{average}\,\{\mathbf{p}(\theta)\mathbf{p}(\theta)^\dagger\}$, for θ in Σ. Taking the eigenvector/eigenvalue decomposition $Q(\Sigma) = WLW^\dagger$, $W^\dagger W = I$, $L = \text{diag}\,\{\lambda_1 \cdots \lambda_n\}$, we define the essential dimension of Σ to be the number k of (essentially) nonzero terms in L. The matrix V has for its columns these k columns of W, so V is N by K and $V^\dagger V = I$.

We derive the SFS method by modifying Capon's method as follows: for each fixed $\theta = \omega$ within Σ we find the linear filter $\mathbf{g} = \mathbf{g}(\omega)$ that minimizes $\mathbf{g}^\dagger R\mathbf{g}$ subject to the constraints $\mathbf{g}^\dagger \mathbf{p}(\omega) = 1$ and $\mathbf{g} = V\mathbf{a}$, where $\mathbf{a}$ is a K by 1 vector and therefore $\mathbf{g}$ is in the span of the columns of V. It is unimportant what $\mathbf{a}$ is; the point is that $\mathbf{g}(\omega)$ cannot now have a sizable component orthogonal to the columns of V. Proceeding as before, we find that the optimal filter is

$$\mathbf{g}(\omega) = V(V^\dagger RV)^{-1}V^\dagger \mathbf{p}(\omega) \times [\mathbf{p}(\omega)^\dagger V(V^\dagger RV)^{-1}V^\dagger \mathbf{p}(\omega)]^{-1} \qquad (10)$$

and the average output power is the SFS estimator:

$$\mathbf{g}^\dagger(\omega)R\mathbf{g}(\omega) = 1/[\mathbf{p}(\omega)^\dagger V(V^\dagger RV)^{-1}V^\dagger \mathbf{p}(\omega)] = \text{SFS}(\omega). \qquad (11)$$

C. Role of normalization in matched-field processing

As they appear in Eqs. (9) and (11), MLM(ω) and SFS(ω) have singularities wherever $p(\omega)$ approaches zero. This is principally a problem near the surface of a waveguide since, from the acoustic boundary conditions, $p(\omega)$ must vanish at the surface. This is easily seen in Eq. (1). Therefore, if one wishes peaks in the ambiguity function to reflect the presence of acoustic sources and not the relative magnitude of the individual $p(\omega)$, then a normalization of the processor is required.

Before deciding what this normalization should be, we must consider the processor response to the noise-only case, to see what background the processor will provide. First, consider the case of white noise, which is totally uncorrelated between the hydrophones so that the time-averaged cross-spectral matrix $R = Q$ is completely diagonal. Further, assume that the average acoustic power delivered to each hydrophone is constant so that R is some multiple of the identity matrix. In this case it is simple to evaluate MLM(ω) from (9):

$$\text{MLM}(\omega) = N(\omega)/\mathbf{p}^\dagger(\omega)\mathbf{p}(\omega), \qquad (12)$$

where $N(\omega)$ is the (as yet undetermined) normalization function. The sector-focused estimator SFS(ω) (11) becomes

$$\text{SFS}(\omega) = N(\omega)/\mathbf{p}^\dagger(\omega)V(V^\dagger V)^{-1}V^\dagger \mathbf{p}(\omega), \qquad (13)$$

$$= N(\omega)/\mathbf{p}^\dagger(\omega)VV^\dagger \mathbf{p}(\omega), \qquad (14)$$

since $V^\dagger V = I$.

Before interpreting what we have just found, let us consider the other extreme type of noise field one might encounter in an acoustic waveguide. Let us consider noise propagating to the array from a large number of distant point sources randomly distributed in depth and range (in fact, distribution in range only is sufficient) so that no near-field effects are included. Then it has been shown that the mode space cross-spectral matrix will be diagonal in form.[23,39,40] If we also assume that the average acoustic power carried by each normal mode is equivalent, then the mode space CSM [i.e., G in Eq. (6)] is simply a multiple of the identity matrix and the phone space matrix is $R = Q = UU^\dagger$. Because $R = UU^\dagger$ is not invertible, we cannot resort to (8) and (9) for the MLM. We must directly solve the problem: minimize $\mathbf{f}^\dagger(\omega)R\,\mathbf{f}(\omega) = \mathbf{f}^\dagger(\omega)UU^\dagger\mathbf{f}(\omega)$, subject to $\mathbf{f}^\dagger(\omega)\mathbf{p}(\omega) = 1$. The MLM esimate will then be $\mathbf{f}^\dagger(\omega)R\,\mathbf{f}(\omega)$ for the optimal $\mathbf{f}(\omega)$. Writing $\mathbf{g} = U^\dagger \mathbf{f}(\omega)$, the problem becomes: minimize $\mathbf{g}^\dagger\mathbf{g}$ subject to $1 = \mathbf{f}^\dagger(\omega)\mathbf{p}(\omega) = \mathbf{f}^\dagger(\omega)U\mathbf{s}(\omega) = \mathbf{g}^\dagger\mathbf{s}(\omega)$. The solution is

$$\mathbf{g} = [\mathbf{s}^\dagger(\omega)\mathbf{s}(\omega)]^{-1}\mathbf{s}(\omega). \qquad (15)$$

From

$$\mathbf{g}^\dagger\mathbf{g} = \mathbf{f}^\dagger(\omega)R\,\mathbf{f}(\omega) = \mathbf{f}^\dagger(\omega)UU^\dagger\mathbf{f}(\omega), \qquad (16)$$

we see that the MLM is $\mathbf{g}^\dagger\mathbf{g}$ or,

$$\text{MLM}(\omega) = N(\omega)/\mathbf{s}^\dagger(\omega)\mathbf{s}(\omega). \qquad (17)$$

Similarly, SFS(ω) takes the form,

$$\text{SFS}(\omega) = N(\omega)/\mathbf{s}^\dagger(\omega)U^\dagger V(V^\dagger UU^\dagger V)^{-1}V^\dagger U\mathbf{s}(\omega). \qquad (18)$$

If the columns of V are simply orthogonalized columns of U then $V = UG$ for some invertible G, and it follows that SFS(ω) reduces to

$$\text{SFS}(\omega) = N(\omega)/\mathbf{s}^\dagger(\omega)\mathbf{s}(\omega). \qquad (19)$$

Now that we know what MLM(ω) and SFS(ω) look like in these noise fields, we turn to the question of the appearance

of their corresponding processor responses.

For the ambiguity surface to be useful, the background against which signals are sought must be relatively flat. If the noise component is white, with the MLM surface for white noise only being given by (12), the best normalization is $N(\omega) = \mathbf{p}(\omega)^{\dagger}\mathbf{p}(\omega)$. However, if the noise is mainly modal, normalizing MLM by $N(\omega) = \mathbf{s}^{\dagger}(\omega)\mathbf{s}(\omega)$ will produce a flatter background. Because we are concerned in this paper with the effects of a sizable modal noise component we have chosen the latter normalization.

III. SIMULATIONS

A. Environmental model

In order to demonstrate the effectiveness of the sector-focused stability technique, we have performed numerical simulations of a shallow-water, range-independent environment, as shown schematically in Fig. 1. This environment, which is modeled after a location on the continental shelf off the southern California coast, consists of a water channel of 500-m depth overlying a sediment layer and an isospeed, semi-infinite subbottom. The sound-speed profile for the water and sediment layers and top section of the subbottom are shown in Fig. 2. In addition to the sound-speed difference, there is also a density contrast between the water layer and the sediment. The water density is 1.0 g/cc and the sediment density is 1.5 g/cc. There is an attenuation factor in the sediment layer of 0.1 dB/λ.

The acoustic field calculations for these simulations were performed using the SACLANTCEN normal-mode acoustic propagation (SNAP) model. For the set of environmental parameters shown in Figs. 1 and 2, the SNAP model indicates 14 propagating modes at a source frequency of 30 Hz. The vertical receiving array consists of 31 equally spaced hydrophones spanning the central 450 m of the water column. Here we will be concerned with propagation over ranges between 10 and 18 km from the source.

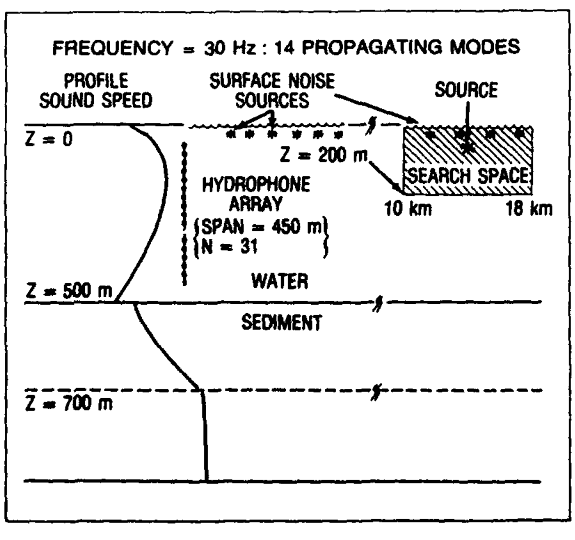

FIG. 1. Schematic diagram showing the acoustic environmental model used in the simulations. The geometrical arrangement of the array, the placement of surface noise sources, and the delineation of the search space (later divided into search sectors) are also indicated in this diagram.

B. Construction of the CSM

The matched-field processor accepts as data input the cross spectral matrix R, which is calculated as the sum of the signal-only matrix S and the noise-only matrix Q,

$$R = S + Q. \tag{20}$$

For a source located at range r and depth z_0, SNAP yields $\mathbf{p}(r,z_0)$ and $\mathbf{s}(r,z_0)$ of Eq. (9), from which S may be calculated,

$$S = \mathbf{p}(r,z_0)\mathbf{p}^{\dagger}(r,z_0). \tag{21}$$

The magnitude of the elements of $\mathbf{p}(r_0,z_0)$ are adjusted to scale the trace of this matrix to give the signal level required for the simulation.

The noise matrix Q has two components. The first is the "modal noise" matrix, representing noise that is correlated between the hydrophones when detected at the array. This is modeled by summing contributions from 1500 individual

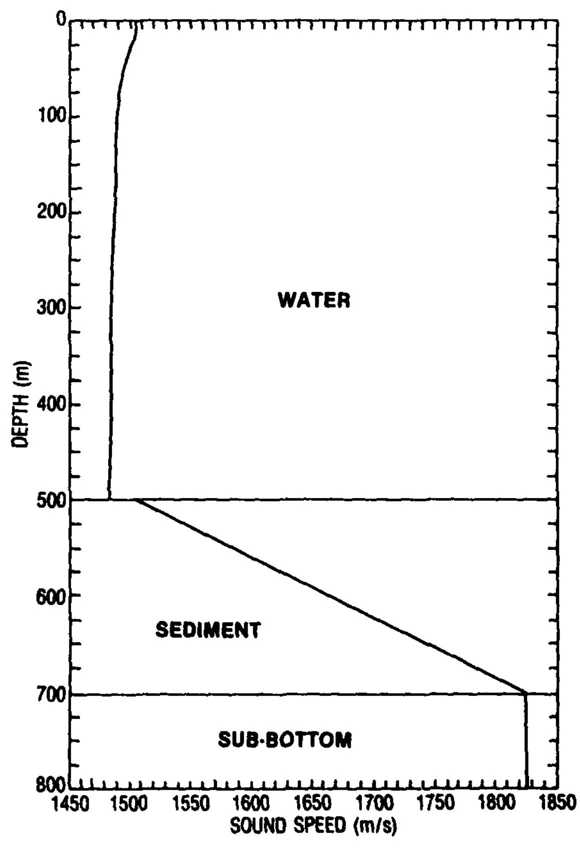

FIG. 2. Sound-speed profile for the continental shelf environment used in the simulations.

sources, which are placed at a depth of 7 m and distributed evenly in ranges from 0.3 to 500 km from the array. The SNAP model implicitly assumes cylindrical symmetry when calculating the acoustic field. Therefore there is a requirement that each of the 1500 sources in the sum represent an annulus of sources surrounding the array whose radius is the range of the source from the array. To accomplish this, the input level of the sources must be scaled so that the average intensity registered at the hydrophones from all of them will be the same. The second component of Q is noise that is completely uncorrelated between the hydrophones ("white noise") and may be modeled as a multiple ρ of the identity matrix. The total noise matrix then becomes

$$Q = \sum_{i=1}^{1500} \mathbf{p}_i \mathbf{p}_i^\dagger + \rho I. \tag{22}$$

In these simulations we will monitor the relative degradation of the MLM and SFS estimators when phase errors are introduced onto the signals received at the hydrophones array. The introduction of these phase errors takes the form of a similarity transformation of R,

$$R' = DRD^\dagger, \tag{23}$$

where $D = \text{diag}\,\{e^{i\theta_j}\}$, and the θ_j are derived from a set of random numbers which are scaled to give the desired maximum phase error incident upon the array. Figure 3 shows a realization of the random set θ_j, scaled to give a maximum value for the error of ± 1 deg. From Eq. (23) we see that the similarity transformation amounts to a rotation of the data matrix about a direction determined by the relative magnitude of the phase errors and through an angle determined by their overall magnitude. Although the perturbations considered here were phase errors, other forms of mismatch would also lead to similar degradation of MLM. The SFS approach offers greater stability to other types of mismatch, such as erroneous environmental parameters, since the exact nature of the perturbations is not important in the development of SFS.

In this paper we use four parameters to characterize the simulated data cross-spectral matrix. First, the source level (SL) is the average of the diagonal elements of the signal-only matrix S, expressed in dB relative to unity. Second, the total noise level (TNL) is defined as the average diagonal element of the total noise matrix Q, also expressed in dB relative to unity. Third, the modal to white noise ratio (MWR) is the ratio of modal to white noise power, again in dB. Finally, the random phase error (RPE) identifies the magnitude of the phase errors, and is expressed as a multiple of the random number elements displayed in Fig. 3. For example, an RPE = 10 deg means that we multiply the phase errors in Fig. 3 by 10, which would give values of θ_j varying between -10 deg and $+10$ deg, and then form the rotation matrix D.

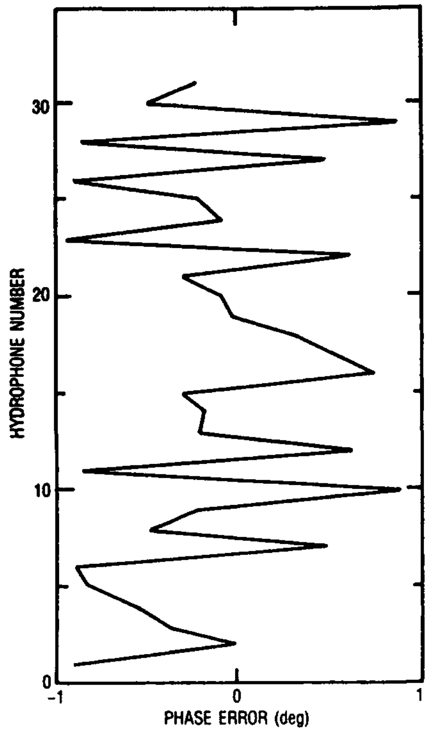

FIG. 3. Realization of the set of random phases (maximum value = ± 1 deg) used to introduce phase errors onto the hydrophone array by means of the similarity transform (23).

C. Comparison of MLM and SFS

Now that we have the simulated data matrix R, the problem is to attempt to extract the source information from it using matched-field techniques. Here we will compare the performance of Capon's MLM to SFS when phase errors are present in the data. Since we are concerned only with stabilizing Capon's MLM we do not include comparisons with conventional (Bartlett) estimation. The advantages and disadvantages of MLM and Bartlett have been discussed at length in the literature.[8,10]

An important input to all matched-field processors is the set of replica field vectors (or steering vectors) $\mathbf{p}(r_i,z_i)$, where i denotes the ith search point (or trial source location). Again, we have used SNAP to produce these vectors for a grid of points within a search space selected to span 0 to 200 m in depth and 10 to 18 m in range (see Fig. 1). Now for each range/depth point in the search grid we can calculate MLM using Eq. (9):

$$\text{MLM}(r_i,z_i) = \frac{\mathbf{s}^\dagger(r_i,z_i)\mathbf{s}(r_i,z_i)}{\mathbf{p}^\dagger(r_i,z_i)R'^{-1}\mathbf{p}(r_i,z_i)}, \tag{24}$$

where we have implemented the $\mathbf{s}^\dagger\mathbf{s}$ normalization of Eq. (17).

When calculating the SFS estimator, one is free to choose the size and shapes of the individual sectors. We leave the question of optimal sector geometry for future investigation. Our aim here is to show that even a very simple choice of sectors yields a very stable processing result. In our simulations of SFS we divided the total search space into 16 equally sized sectors. Each sector covers 50 m in depth and 2 km in range. Thus, calculation of the sector-focused proces-

sor requires the formation of 16 matrices V describing each of the 16 sectors within the search space.

For each sector Σ, the calculation of V proceeds in three main steps. First we form a cross-spectral matrix as follows:

$$Q(\Sigma) = \sum_{\text{many sources}} \mathbf{p}_j \mathbf{p}_j^{\dagger}, \tag{25}$$

where the sum is taken over many sources distributed throughout the sector of interest. The exact number of sources in this sum is not important so long as it is large compared to the number of hydrophones in the receiving array. This assures that the eigenvectors of N will span the sector in question.

Next, we numerically determine the eigenvalues and eigenvectors of $Q(\Sigma)$. The columns of V are then chosen to be a set of K eigenvectors corresponding to the largest (not close to zero) eigenvalues of $Q(\Sigma)$. The exact value of K and the form of V depends, in general, on both the size and location of the sector in question. However, K is limited to take a value between one and the number of propagating modes in the waveguide. This is easily demonstrated by considering two limiting cases. First, consider shrinking the size of the sector to a single point. In this case $Q(\Sigma)$ is composed of a single dyad and will therefore have exactly one nonzero eigenvalue. Second, expand the sector to span the entire waveguide. Here the number of large eigenvectors will be equal to the number of degrees of freedom. For an acoustic waveguide this is equal to the number of modes, and so the matrix $Q(\Sigma)$ will have M nonzero eigenvalues. In our simulations we choose $K = 11$, so that V is a 31×11 matrix.

Within each sector, we can now form the SFS estimator as follows:

$$\mathrm{SFS}(r_i,z_i) = \frac{\mathbf{s}^{\dagger}(r_i,z_i)\mathbf{s}(r_i,z_i)}{\mathbf{p}^{\dagger}(r_i,z_i)V(V^{\dagger}R'V)^{-1}V^{\dagger}\mathbf{p}(r_i,z_i)}. \tag{26}$$

D. Quantification of the estimator functions

In order to quantify the performance of the estimators, and to distinguish between visually similar ambiguity surfaces, we calculate a detection factor for each function as follows:

Detection Factor: $\mathrm{PBR} = (P - \mu)/\sigma$(expressed in dB), (27)

where P is the value of the ambiguity function at the known source location, μ is the average level of the ambiguity function ignoring the region immediately around the source peak, and σ is the standard deviation of the ambiguity function ignoring the region immediately around the source peak.

This factor has been discussed previously in the literature.[13] We do not infer statistical behavior from it, but use it as a measure of "goodness."

E. Results and analysis

In all of the simulations considered here we maintain constant values for: the signal level (SL = 50 dB); the total noise level (TNL = 60 dB); and the modal-to-white noise ratio (MWR = 30 dB). We also consider a single source located at a range of 14.5 km and a depth of 30 m. We will be concerned with the performance of the MLM and SFS estimators as a function only of the random phase error introduced at the hydrophone array. Essentially, the requirement here is to localize a source with input level -10 dB within a noise field dominated by spatially correlated (modal) noise.

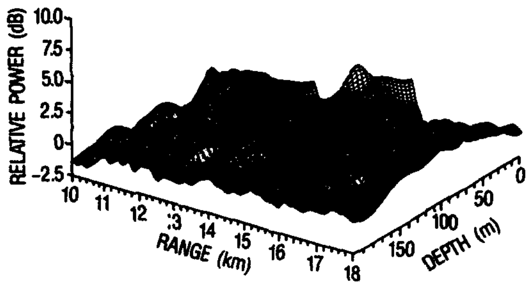

FIG. 4. MLM ambiguity surface with a random phase error of 0 deg.

In Fig. 4 we see the MLM ambiguity surface for an RPE = 0 deg. On this surface the signal peak is clearly resolvable against a moderate background with a detection factor PBR = 9.06 dB, and is located at the correct range and depth. This is to be expected, since there is no difference between the replica field vectors and those used to construct the data matrix. In Fig. 5 we see the SFS ambiguity surface, also for the case of zero phase error. This surface was produced by using the division of the search region into 16 sectors, each 50 m in depth by 2 km in range, described above. This figure indicates that, in the RPE = 0 deg case, the SFS estimator offers no advantage over MLM. Although the signal peak is still easily resolvable and correctly located, the background is actually slightly worse than with MLM. The value of PBR falls to 8.28 dB.

Let us now see the effects of increasing the phase error. In Fig. 6 we display the MLM ambiguity surface for an RPE = 45 deg. It is seen that, with this amount of phase

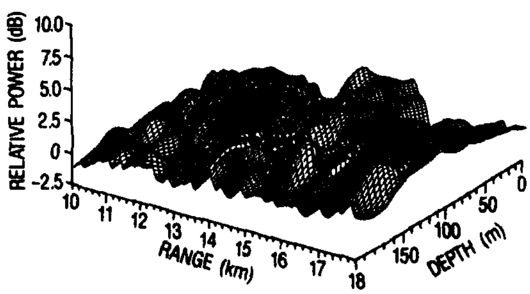

FIG. 5. SFS ambiguity surface with a random phase error of 0 deg.

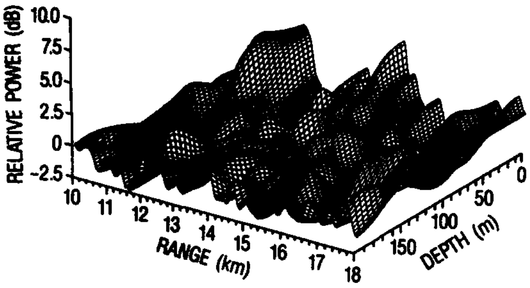

FIG. 6. MLM ambiguity surface with a random phase error of 45 deg.

error, the MLM processor has been degraded and distorted to the point where the signal peak has vanished and has been replaced by two large spurious sidelobes. One of these is located at range 11.5 km while the other, although found at range 14.5 km, is located at the surface rather than at the input depth of 30 m. Evidently the MLM processor is ineffective for localization purposes once this level of phase error is reached. In contrast, we see the SFS ambiguity surface for an RPE = 45 deg in Fig. 7. Here the signal peak is still clearly resolvable above the background. In fact, comparison of Fig. 7 with Fig. 5 shows that the introduction of phase error has led to no discernible degradation or distortion of the SFS surface. Rather, the quality of the surface, from a signal localization viewpoint, seems to have *improved* slightly, with the value of PBR increasing to 8.55 dB when the RPE is 45 deg.

In Fig. 8 curves are plotted that indicate the variation of PBR for both the MLM and SFS processors as a function of RPE. As the phase error is increased we see the MLM curve fall very steeply from its initial value of 9.06 dB at RPE = 0 deg, to a value of 2.55 dB at RPE = 20 deg. It then decreases more slowly as the RPE is further increased, reaching 1.81 dB at RPE = 100 deg. However, the SFS curve is dramatically different. As the phase error is increased, the value of PBR also increases (see again Figs. 5 and 7), reaching its maximum of 8.55 dB at RPE = 45 deg. After this point, the values of PBR begin to slowly fall; but the processor still

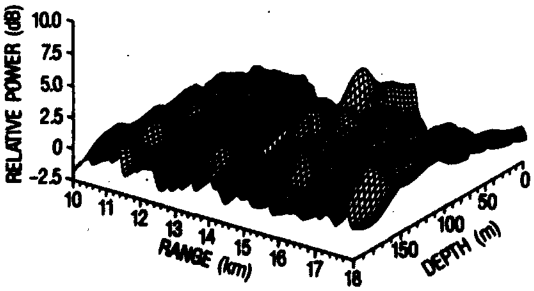

FIG. 7. SFS ambiguity surface with a random phase error of 45 deg.

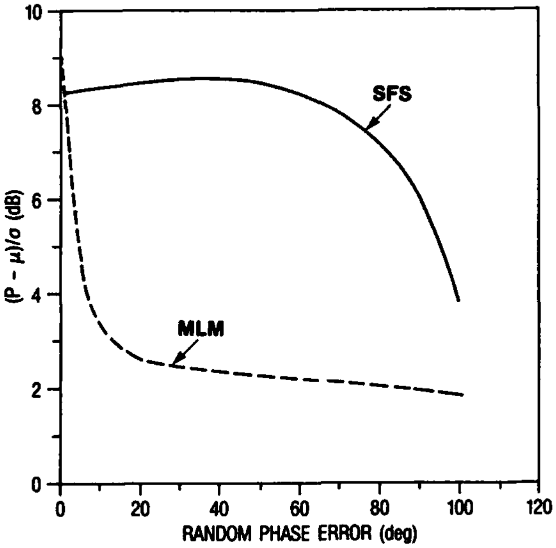

FIG. 8. Variation of PBR for both the MLM and SFS processors as a function of random phase error.

performs effectively out to relatively large values of RPE, giving a PBR of 5.99 dB at RPE = 90 deg and a PBR of 3.84 dB at RPE = 100 deg. Apart from a narrow range near RPE = 0 deg (where both processors work almost identically) the SFS consistently outperforms the MLM by 2–6 dB over the range of RPE considered.

IV. CONCLUSIONS

An improved matched-field processor, the "sector-focused stability" (SFS) method, has been developed to provide more stable source estimation performance in a waveguide in the presence of high levels of correlated (modal) noise. Under these conditions the "maximum likelihood" (ML) method shows serious deterioration when small random phase errors are introduced onto the hydrophones. However, the SFS method continues to return stable and accurate source estimates with levels of random phase error which are several times that which induces failure of the ML method.

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